

18. Solve $\nabla^2 u = 0$, $0 < x < a$, $0 \leq y < b$
 $u(x, 0) = f(x)$, $0 \leq x \leq a$, $u(x, b) = 0$, $0 \leq x \leq b$
 $u_x(0, y) = u_x(a, y) = 0$ using separation of variable.
19. Obtain the solution Dirichlet problem involving Poisson equation.
20. Explain the method of eigen function to obtain Green's function.

APRIL/MAY 2025

23PMA23 — PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours

Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.



1. Write the neat conduction equation.
2. Classify the partial differential equation $u_{xx} + x^2 u_{yy} = 0$.
3. Define Cauchy problem.
4. Write d'Alembert solution of the cauchy problem for one dimensional wave equation.
5. Define kinetic energy.
6. Define angular frequency.
7. Define boundary value problem.
8. Define Neumann problem.

9. Define Dirac delta function.

10. Write Green's function in three dimension.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Derive one dimensional wave equation.

Or

(b) Find the general solution of
 $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 0$.

12. (a) Solve the following cauchy problem of an infinite string with the initial condition

$$u_{tt} - c^2 u_{xx} = 0, x \in R, t > 0$$

$$u(x, 0) = f(x), x \in R$$

$$u_t(x, 0) = g(x), x \in R$$

Or

(b) Determine the solution of the cauchy's Goursat problem

$$u_{tt} = c^2 u_{xx}, u(x, t) = f(x) \text{ on } x + ct = 0$$

$$u(x, t) = g(x) \text{ on } x - ct = 0$$

where $f(0) = g(0)$.

13. (a) State and prove uniqueness theorem.

Or

(b) Solve the following initial boundary value problem using the method of separation of variables.

$$u_t = k u_{xx}, 0 < x < l, t > 0$$

$$u(0, t) = 0, t \geq 0$$

$$u(l, t) = 0, t \geq 0$$

$$u(x, 0) = f(x), 0 \leq x \leq l$$

14. (a) State and prove continuity theorem.

Or

(b) State and prove mean value theorem.

15. (a) Prove that Green's function is symmetric.

Or

(b) Explain the method of images to obtain the Green's function.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive the equation of vibrating membrane.

17. Obtain the solution of linear hyperbolic equation using Riemann's method.

